

# Probing the Compositionality of Intuitive Functions

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### How do people make sense of complex functional structure?

- Many psychological tasks involve learning and representing intuitive functions
- But how do people represent complex functional structure?
- Do they compose structural patterns into simpler parts?

### Gaussian Process Regression as a Theory of Intuitive Functions

- $\mathcal{GP}$  as a distribution over intuitive functions  $f \sim \mathcal{GP}(m, k)$  (following Lucas et al., 2015)
- $m(\mathbf{x}) = \mathbb{E}[f(\mathbf{x})]$
- $k(\mathbf{x}, \mathbf{x}') = \mathbb{E}[(f(\mathbf{x}) - m(\mathbf{x}))(f(\mathbf{x}') - m(\mathbf{x}'))]$

### Three Theories of Functional Structure Learning

- Agnostic Structure Learning: Radial basis function**
  - When will it finally be the Last Christmas? (Graph)
  - Achieves great interpolation (Graph)
  - Terrible at extrapolation (Graph)
- Implicit Structure Learning: Spectral density mixture (Wilson & Adams, 2013)**
  - When will it finally be the Last Christmas? (Graph)
  - Achieves great interpolation (Graph)
  - Mediocre at extrapolation (Graph)
- Explicit Structure Learning: Compositional structure search (Duvenaud et al., 2013)**
  - When will it finally be the Last Christmas? (Graph)
  - Finds: Lin + Per x RBF (Graph)
  - Sensible extrapolations (Graph)

- Building blocks: RBF, Linear and Periodic
- Rules: Addition and Multiplication
- Let's put them to a psychological test!

### Pattern Completion

- Which of the three proposals best completes the pattern? (Three graphs showing different completions of a partial wave)
- Compositional and spectral mixture ground truth: (Two bar charts showing 'Proportion of choices' for Compositional, RBF Kernel, and Spectral Kernel)

### Manual Pattern Completion

- Draw predictive line into dots sampled from a compositional kernel (Three graphs showing manual completions)
- Results for inter- and extrapolation: (Two bar charts showing RMSE for Interpolation and Extrapolation across Compositional, RBF Kernel, and Spectral Kernel)
- Compositional structure search especially good for extrapolations

### MCMC with People

- Choose between current completion and proposal
- Compositional ground truth: (Three bar charts showing 'Accepted kernels' for Linear + Periodic, Linear x Periodic, and Linear + Periodic x Radial Basis)
- Match with compositional structure search:  $\rho = 0.91, p < 0.01$
- Real word data: (Two bar charts for 'Airline Passengers' and 'Gum Memberships' showing 'Accepted kernels')
- Match with compositional structure search:  $\rho = 0.83, p < 0.01$

### Predictability Assessment

- Assess how well you could predict this function (Schulz et al., 2015) (Three graphs showing function prediction tasks)
- Single and comparative judgements (Two line graphs: 'Predictability' and 'Direct Comparison' showing Mean Judgement vs Sample Size)
- Compositional functions assessed as more predictable

### Conclusion & Outlook

- Intuitive functions exhibit compositional inductive biases
- This is good news as compositionality leads to rapid model building
- Classic paradigms, chunking, change detection

### References

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