Probing the Compositionality of Intuitive Functions

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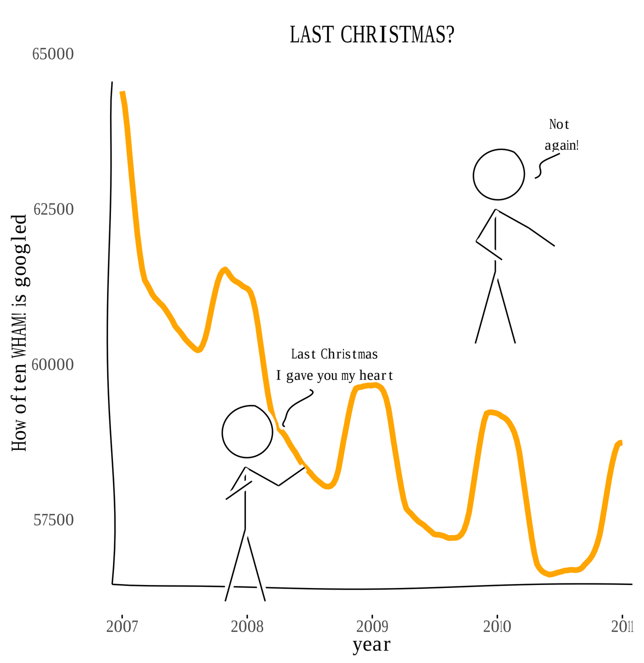
How do people make sense of complex functional structure?



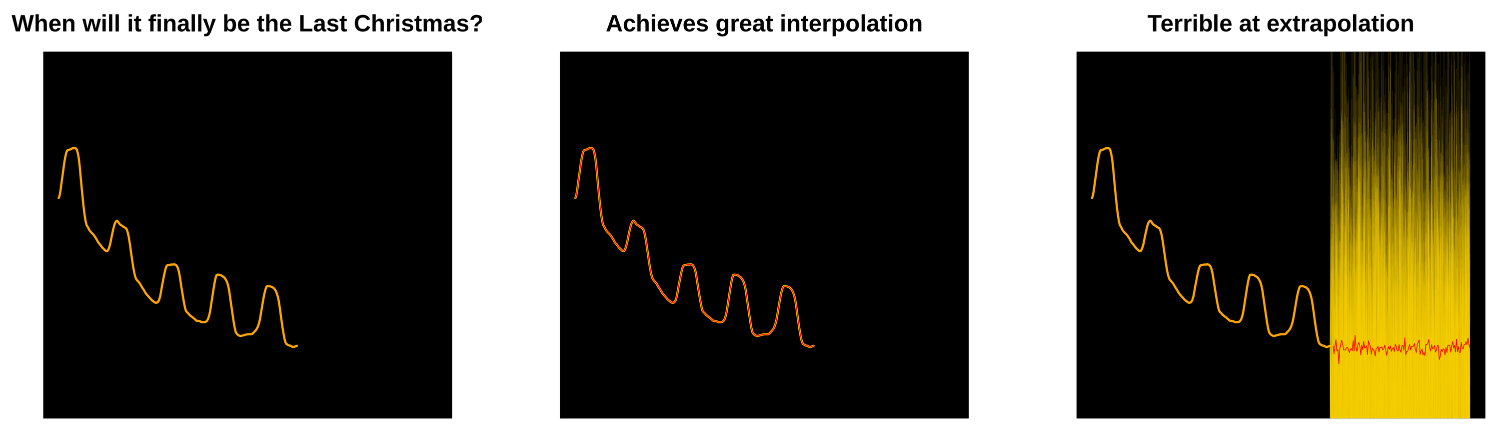
- Many psychological tasks involve learning and representing intuitive functions
- But how do people represent complex functional structure?
- Do they compose structural patterns into simpler parts?

Gaussian Process Regression as a Theory of Intuitive Functions

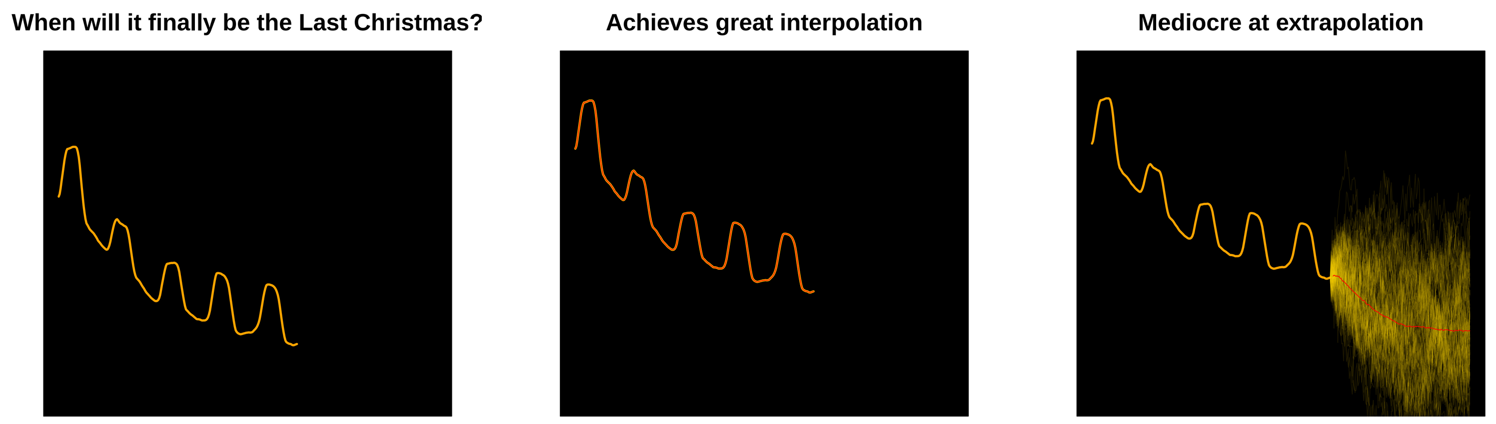
- $\mathcal{GP}$ as a distribution over intuitive functions $f \sim \mathcal{GP}(m, k)$ (following Lucas et al., 2015)
- $m(\mathbf{x}) = \mathbb{E}[f(\mathbf{x})]$
- $k(\mathbf{x}, \mathbf{x}') = \mathbb{E}\left[(f(\mathbf{x}) - m(\mathbf{x}))(f(\mathbf{x}') - m(\mathbf{x}'))\right]$

Three Theories of Functional Structure Learning

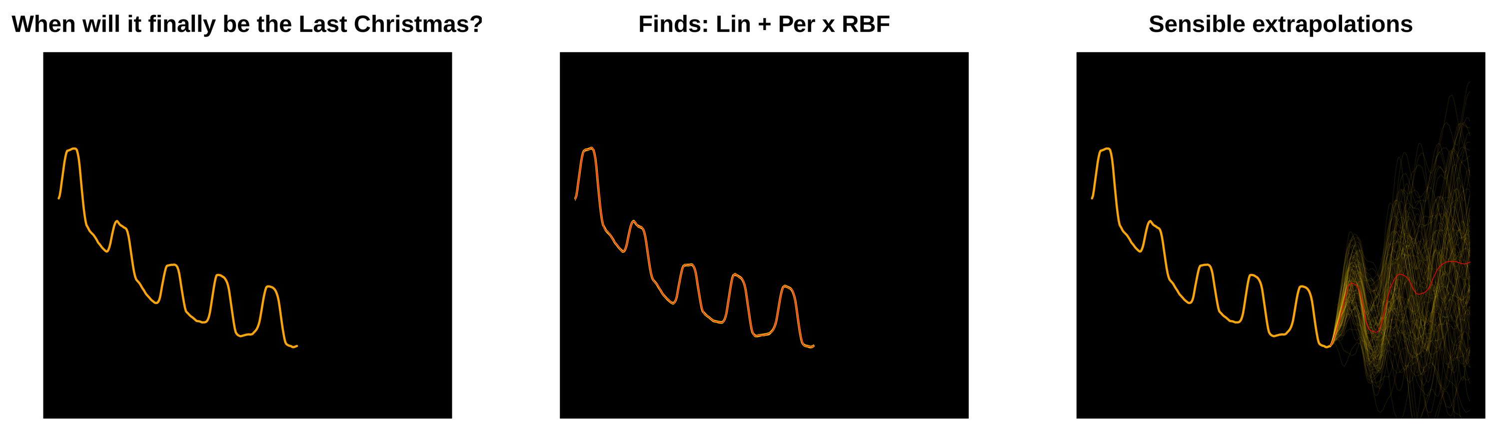
- Agnostic Structure Learning: Radial basis function



- Implicit Structure Learning: Spectral density mixture (Wilson & Adams, 2013)



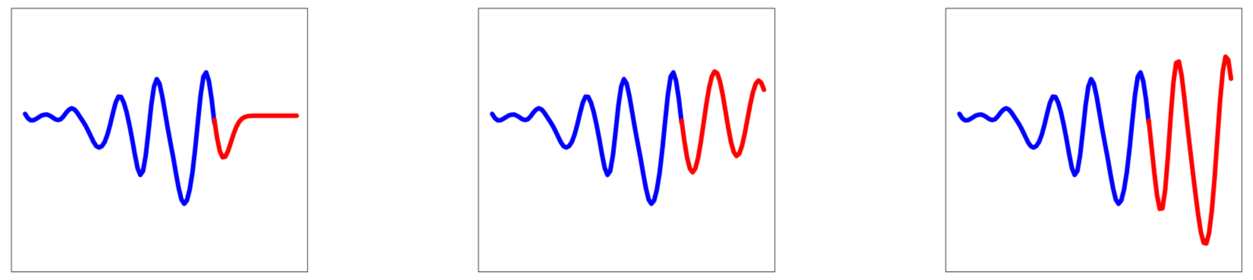
- Explicit Structure Learning: Compositional structure search (Duvenaud et al., 2013)



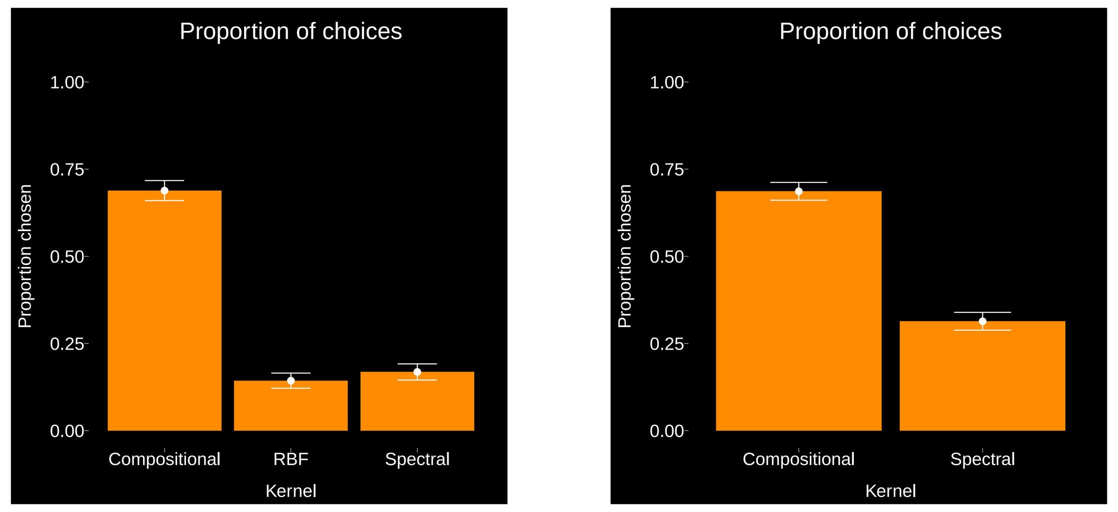
- Building blocks: RBF, Linear and Periodic
- Rules: Addition and Multiplication
- Let's put them to a psychological test!

Pattern Completion

- Which of the three proposals best completes the pattern?

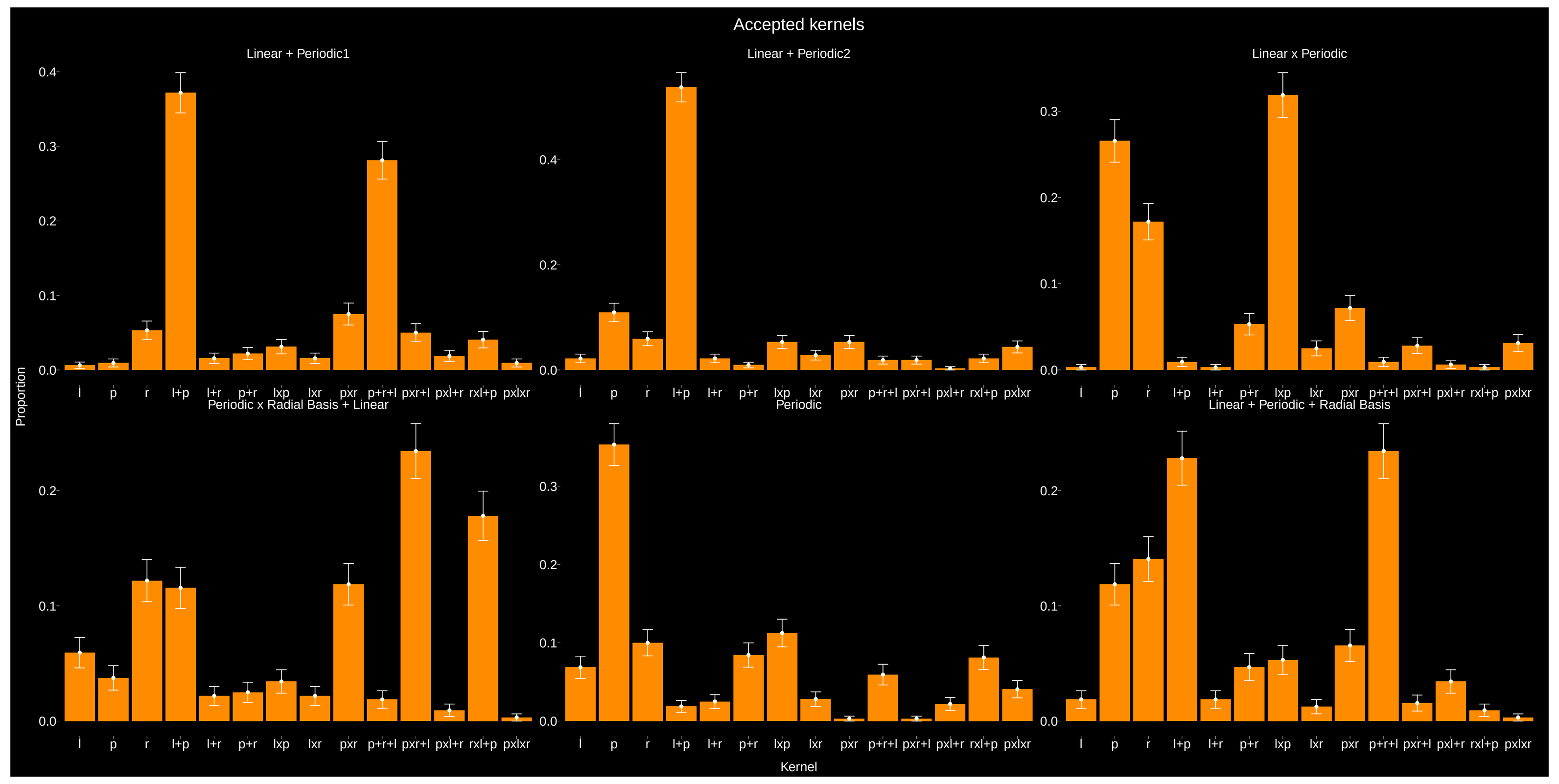


- Compositional and spectral mixture ground truth:

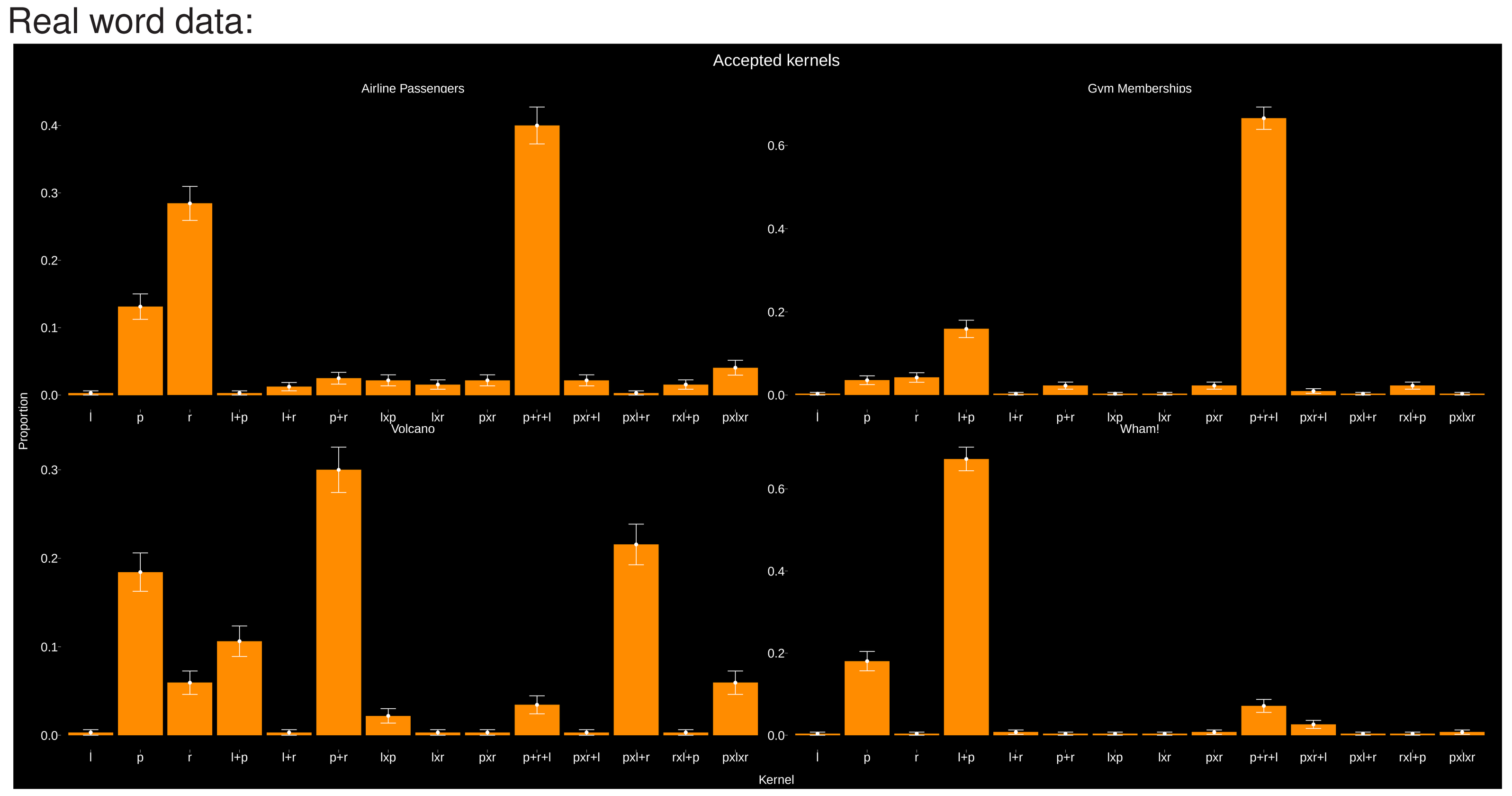


MCMC with People

- Choose between current completion and proposal
- Compositional ground truth:



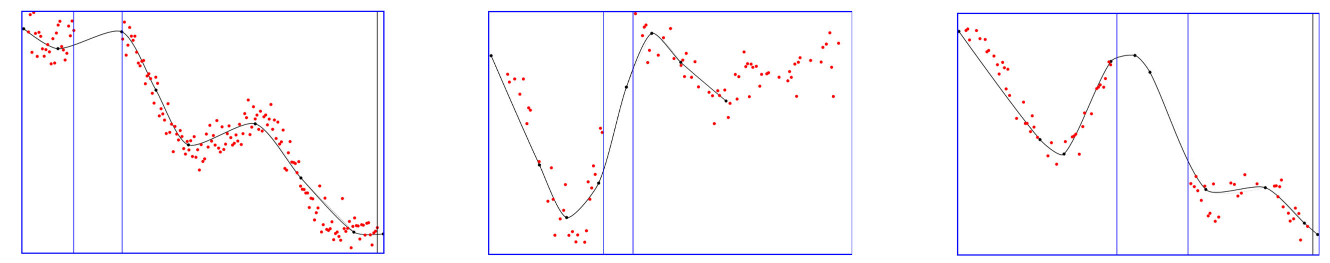
- Match with compositional structure search: $\rho = 0.91$, $p < 0.01$
- Real word data:



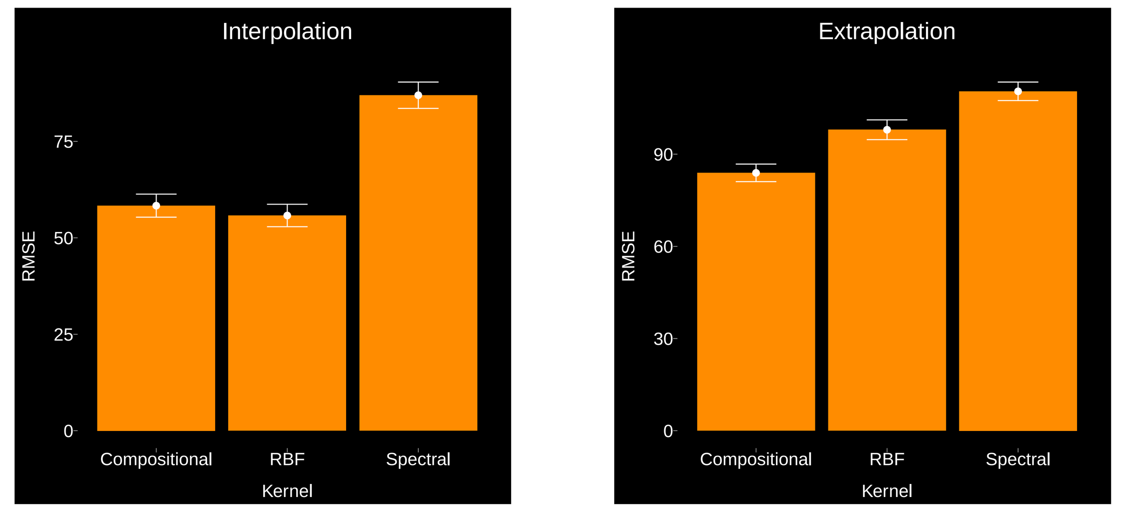
- Match with compositional structure search: $\rho = 0.83$, $p < 0.01$

Manual Pattern Completion

- Draw predictive line into dots sampled from a compositional kernel



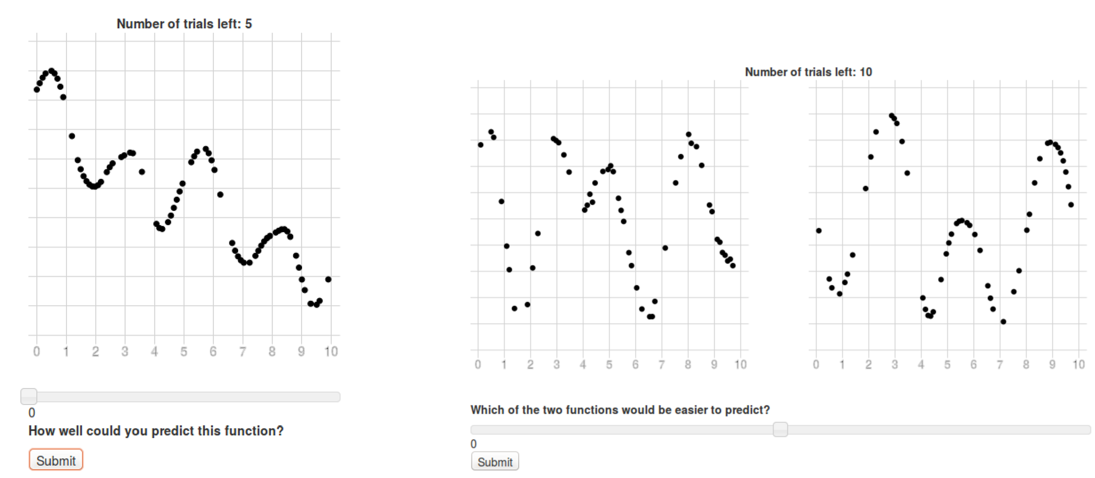
- Results for inter- and extrapolation:



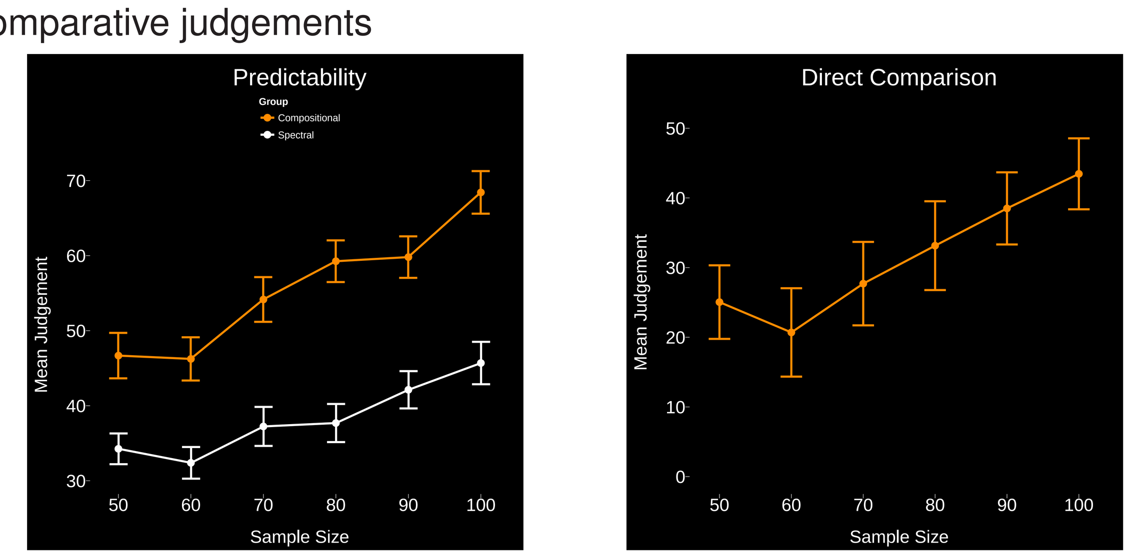
- Compositional structure search especially good for extrapolations

Predictability Assessment

- Assess how well you could predict this function (Schulz et al., 2015)



- Single and comparative judgements



- Compositional functions assessed as more predictable

Conclusion & Outlook

- Intuitive functions exhibit compositional inductive biases
- This is good news as compositionality leads to rapid model building
- Classic paradigms, chunking, change detection

References

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